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8 SOLIDARITY AND SOCIAL NETWORKS

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1. INTRODUCTION

"The study of solidarity thus grows out of sociology."
– Emile Durkheim (1893, p. 67)

In their agenda of problems with respect to the theory of solidarity, Fararo and Doreian (1998) have formulated a call and a challenge to mathematical sociology to situate itself at the core of the sociological enterprise. What is taken as problematic in the foundational paper of Fararo and Doreian is "the very 'groupiness' of the group itself" (1998, p. 6). These writers accord prominence, substantively and analytically, to networks of interaction. Moreover, they call for models that "lift" these networks to "what is, for the actors, a higher-order social entity" (p. 6). At least as one major component of their agenda, they call for models and analyses of the group as an entity that emerges from social flux to be characterized dually as a social network with a structural form and as a higher-order social entity that transcends its individual members (p. 6).

As theorists of solidarity often do, Fararo and Doreian introduce their review of sociological approaches by mentioning Durkheim. In this paper we spend more time with this progenitor of modern sociology, and in particular with his 1893 work, *The Division of Labor in Society*, aptly described by LaCapra (1985, p. 79) as now "referred to with the *pro forma* awe which scholars reserve for recognized classics, but to which little real reference is made in the analysis of

problems." We aim to end such neglect of Durkheim's relevance for social network analysis, much as Alexander (1988) has championed his centrality for cultural studies (see also footnote 5). We agree with Collins (1994, p. 236) that the Durkheimian tradition "remains somewhat of a half-kept secret amidst the various empirical and theoretical proceedings of the field today," despite the fact that its potential for "pulling sociology together around a common core" remains "even more powerful today than ever before."

This paper begins, then, with a discussion of Durkheim's understanding of the problem of solidarity and its implications for formal modeling. We focus on four aspects: the morphology of social association, collective representation as an emergent property of groups, the analytical tension that Durkheim insists on bringing to bear in identifying both the opposition and the unity of individuals and groups, and the implications of types of solidarity for types of exchange.[1]

In Section 3 of the paper we bring these considerations to bear in the formulation of a new interpretive context for one family of log-linear models for networks of interaction frequency, namely: models of quasi-symmetry and of skew-symmetry. All the technical results that we need are already known (Caussinus, 1965; Goodman, 1985; Sobel, Hout and Duncan, 1985; Yamaguchi, 1990). Their application in the formulation of network-analytic models for square tables of interaction frequency is novel. In particular, our framework postulates, and therefore allows the test of, a unidimensional collective representation of group hierarchy as coexisting with a network of social relations, the reduced-form structure of which is itself subject to representation and testing within the same model. Moreover, the degree to which specific individuals take part in group life can be measured, and inferences can be tested.

The data context for Section 4 is interagreement among U.S. Supreme Court justices during the first four terms of the "Rehnquist Court" (Segal and Spaeth, 1993). Several features of this network are incorporated within a single modeling context: detailed patterns of agreement, reduced-form "block" structure, the emergence of person-specific "strengths" as a single dimension partially governing the detailed interactions, and a second dimension pertaining to the degree to which actors can stand apart from the ideological structuring and weave coalitions together. This modeling context for group solidarity thus allows incorporation of multiple principles of social order that should have wider applicability to analyses of square networks of counted data.

One limitation of the work reported here should be identified at the outset. Fararo and Doreian call for models that bridge structure and process; however, models in this paper, while relevant for process outcomes, are purely structural. We claim to make progress in articulating relations between social networks and emergent collective properties, but we have made no direct progress in combining process and structural models.

2. DURKHEIM ON SOLIDARITY

Is friendship based on resemblance or on dissimilarity among friends? This question is posed in the opening chapter of Durkheim's 1893 work, *The Division of Labor in Society*. After reviewing the various positions taken by classical thinkers (Aristotle, Euripides, Heraclitus) and by modern authors on this question, Durkheim concludes that "we seek in our friends the qualities we lack, since in joining with them, we participate in some measure in their nature and feel less incomplete" (1893, pp. 55–6). It is "this division of labor, which determines the relation of friendship." In fact, the "true function" of the division of labor is "to create in two or more persons a feeling of solidarity." It follows that, contrary to analysts who see the division of labor in rational, economic terms, it is necessary to consider the division of labor "in a new light," since "the economic services that it can render are picayune compared to the moral effect that it produces" (p. 56).

It seems as if Durkheim requires three actors in order to discuss two friends: the two individuals and the collective unit that they form with their interpersonal ties of exchange or sharing across several domains of activity. Indeed, Alpert (1941, p. 174) argued that the term *solidarité* as Durkheim used it in the 1890s in France referred, in an objective and even biological sense, to a type of relation between a whole and its parts, and he suggested "cohesion" as a possible synonym. Here in Durkheim's discussion of friendship we have the "membership relation linking two levels of structure" that is labeled a "solidary tie" by Fararo and Doreian (1998). For Durkheim an important quality of this tie is its variable expression in concrete social forms; "it is not the same in the family and in political societies; we are not attached to our country in the same fashion as the Roman was to his city or the German to his tribe" (p. 66).

It follows for Durkheim that "the study of solidarity thus grows out of sociology" (p. 67). Durkheim felt that the economists, along with the "moralists and psychologists," had made little progress in the

scientific study of solidarity owing to their common analytical focus on what he referred to with evident scorn as "independent individuals, the zero of social life" (1893, p. 179 n. 12), and on the possible motivations of such imaginary individuals to seek to be part of a group. Such analysts thereby eliminated from the phenomenon "all that is peculiarly social in order to retain only the psychological germ whence it developed" (p. 67).[2]

What Durkheim has to contribute to a discussion of the mathematical formalization of the concept of solidarity, we argue, is not so much the contents of his theory of solidarity and the division of labor, but rather the envelope of important analytical concerns that he brought to bear in his analysis. In addition to his admonition (already discussed) to begin with the social rather than with the individual, we will focus on four types of concern that Durkheim elaborated: social morphology, collective representation of the group, the strain between individualizing and collective group tendencies, and implications of solidarity for social exchange.

The first constellation of concerns, social morphology, pertains to what we might today call the reduced-form, or ideal-typical representation of complex networks. In his Latin thesis, Durkheim traced his interest in morphology to that of Montesquieu, who distinguished types of societies "differ[ing] in the number, arrangement, and cohesion of their component parts.... Moreover, the structures of these societies are not the same, nor are their members united by the same ties" (Durkheim, 1892, pp. 26–27). Montesquieu had identified various types of society, such as monarchy, aristocracy, and republic. Durkheim pointed out (p. 32) that Montesquieu had distinguished them and named them "not on the basis of division of labor or the nature of their social ties, but solely according to the nature of their sovereign authority." Durkheim criticized this as a failure to see "that the essential is not the number of persons subject to the same authority, but the number bound by some form of relationship" (p. 38).[3] As to reduced-form representation, Durkheim observed for example that in a republic the city-state appears to be "a kind of block made up of homogeneous components, none superior to the others" (p. 27). "Block" is a translation of the Latin word *molis* that Durkheim used; this molar or "block-like" character of elementary social units is just what Durkheim went on to call, in his French-language thesis, "mechanical solidarity." Structures characterized by mechanical solidarity "are coherent, not in spite of their homogeneity, but because of their homogeneity" (1893, p. 179).

If earlier and simpler forms of social life were in Durkheim's mind characterized by the mechanical solidary of homogeneous "blocks," then more complex and contemporary social structure required for Durkheim a representation that perhaps comes closer to the "block-models" of White and collaborators (White, 1992; Breiger, 1991), Burt (1992), and Faust and Wasserman (1992), which models emphasize ideal-typical relations among the units.[4] Today according to Durkheim the direct bond is the one of interdependence among the units; direct subsumption of the individual within a social group is attenuated (Alpert, 1941, p. 177). In modern societies each individual draws "a profound distinction between himself and society," hence "social solidarity cannot be the same" as in earlier times; today solidarity "results from the division of labor, which makes the citizen and the social orders dependent upon each other" (Durkheim, 1892, pp. 33–34). The unity of the social organism is "as great as the individuation of the parts is more marked," and "because of this analogy, we... call solidarity which is due to the division of labor, organic" (1893, p. 131).

Seemingly in an anticipation of Granovetter's (1973) thesis on the strength of ties that is no less than startling to a contemporary network analyst, Durkheim's theory about the transition from mechanical to organic solidarity rests on the weakness of the strong ties characteristic of mechanical solidarity. "What truly measures the relative force of two social ties is the unequal facility with which they break down" (1893, p. 148). Where solidarity rests on resemblances, breaks are more frequent and easier to bring about; members of nomadic clans simply abandon their chief when they find his authority too oppressive. In such societies the individual "contains within himself all that social life consists of," and so "he can go and carry it elsewhere." Therefore, "it may appear astonishing that a tie which binds the individual to the community by absorbing him into it can be broken or made with such facility. But what makes a social tie rigid is not what gives it resistive force" (p. 151). Under a regime of mechanical solidarity, precisely because social work is very little divided, society can lose a certain number of its members "without the economy of its internal life being disturbed," and so does not strongly oppose these departures (p. 151). The evolutionary argument is thus based on the assumption that "all social links which result from likeness progressively slacken" (p. 173) to the extent that "there are more individuals sufficiently in contact to be able to act and react upon one another." The phrase Durkheim proposes for this development is "dynamic or moral density" (p. 257),

one cause for example being the concentration in space of disparate groups; Durkheim recognized that "the union of groups is also accompanied by interstitial growth" and multiplication of relations (p. 257). Thus, although according to common sense "the ties which result from the division of labor, while being more numerous, would be weaker than the others,... the contrary is the truth" (p. 148).

A second major set of issues of Durkheim's that we believe is pertinent to the formulation of models of solidarity concerns emergent properties of groups and, in particular, collective representations that both issue from and regulate interaction within groups. Much of *The Division of Labor* is taken up in expounding the argument that each type of solidarity, mechanical and organic, is associated with the emergence of distinctive systems of legal rules – repressive and restitutive, respectively, in rough analogy to primitive criminal law in comparison to the law of contracts – which "impose uniform beliefs and practices upon all" (1893, p. 226).

Mechanical solidarity can be consistent with a "unilateral" structuring of authority, such as that of a father of a Roman family with his children (p. 180). Indeed, one of the most astute readers of Durkheim draws a connection between ritualized interactions and the emergence of a hierarchical or power dimension based on location with respect to "sacred objects" (Collins, 1988, p. 114). In contrast, even under a complex division of labor, society "does not become a jumble of juxtaposed atoms;" rather, the members are united by "ties which extend deeper and far beyond the short moments during which the exchange is made." In fact, "from the nature of the chosen task permanent duties arise" (1893, p. 227).

In all of the above-mentioned cases, the specific structuring of the group is responsible for the quality of emergent law, morality, and regulation. The emergent representation that the group comes to hold of itself "must be sought chiefly in the way in which the associated individuals are grouped" (quoted by Lukes, 1973, p. 231). Rules, habits and daily routines do not "create the state of mutual dependence in which solidary organs find themselves, but only express in clear-cut fashion the result of a given situation" (1893, p. 366). In a manner that again recalls Montesquieu, as Lukes points out, Durkheim stipulates that society is comprised not only of a material side but a "spirit," a soul, which is "nothing other than a complex of ideas" which transcend the individual and "come into being and sustain themselves only through the interaction of a plurality of associated individuals" (quoted by Lukes, p. 131). Thus does morphology lead to an emergent

property: collectively held conceptions of the group, its stratification system, and its morality, especially insofar as the last-mentioned term refers to "attachment to social groups" (quoted by Lukes, p. 113).[5]

The third set of problems on which Durkheim focused and which we believe are productive for insight with regard to the formulation of models of solidarity concern strains between individualizing and collective group tendencies. In particular, in his essay on the dualism of human nature and its social conditions, Durkheim defines "duality" as "the double existence that we lead concurrently: the one purely individual and rooted in our organisms, the other social and nothing but an extension of society" (1914, p. 162; see also Breiger, 1990 on relations between Durkheim's conception of duality and that of his contemporary, Georg Simmel). The imbalances that lead to suicide pertain to disproportions in the ratio of strengths of collective versus individual forces. Too much society leads to insufficient development of individualism and, thus, to pathologies of altruism (such as the suicide of women upon the death of their husbands; here suicide is not a right but a duty). On the contrary, too little society leads to a conditions of normlessness and egoism that also render the problem of solidarity insoluble, giving rise to a person who is "a mystery to himself, unable to escape the exasperating and agonizing question: to what purpose?" (quoted by Lukes, p. 209).

In *The Division of Labor*, Durkheim defined "negative solidarity" as relations that link mere things (such as personal property) to persons, rather than persons to each other (1893, p. 116). Positive solidarity binds the individual to society directly, without any intermediary. Under regimes of mechanical solidarity, characterized by positive solidarity, "this solidarity can grow only in inverse ratio to personality" (p. 129). Regimes of organic solidarity in contrast call for the development of individual personalities, as the person depends upon others "in the same measure that he is distinguished from them, and consequently upon the society which results from their union" (p. 226). At issue here is the extent with which individuals are constrained to accept a common group life, and the corollary extent to which they can exercise their own decisions about the degree of their participation in a common collective life.

The fourth set of concerns that motivate our interest in formalizing Durkheim's ideas on solidarity pertain to social exchange. While Durkheimian, this emphasis on exchange was not a primary focus of his. Although Durkheim was not by any means an "exchange theorist," one of his notable intellectual descendants was. Lévi-Strauss likened

mechanical solidarity to restricted exchange and organic solidarity to generalized exchange (though he subsequently backed away a bit from this analogy; see Ekeh, 1974, p. 61). Restricted exchange for Lévi-Strauss refers to "mutual reciprocities limited to two partners" (Ekeh, p. 50). Generalized exchange refers to reciprocities involving at least three actors. The difference is that "in generalized exchange no party gives to the party from whom he receives" (p. 50); thus, the two types of exchange differ qualitatively, not quantitatively. With respect to three persons, generalized exchange might operate as follows: $A \rightarrow B \rightarrow C \rightarrow A$, where "$\rightarrow$" signifies "gives to" (Ekeh, p. 50). However, a large number of close variants is possible (Ekeh, pp. 52–55).

Restricted social exchange relationships are "brittle in nature" (Ekeh, p. 52). With respect to the analysis we sketched above of Durkheim as a theorist of the weakness of strong ties, we may agree with Ekeh (p. 52) that a system of restricted exchange "is marked by mechanical solidarity." As a corollary, Ekeh (p. 74) observes that under a regime of organic solidarity integrative mechanisms are increasingly required to the extent that differentiation increases; thus he suggests "that ideally the growth of this differentiation is within a cultural matrix infused with a morality of generalized exchange."

Having identified four sets of problems that concern us in Durkheim's theory of solidarity, we will now consider specific models.

3. MODELS OF SOLIDARITY

Maintaining our focus on types of social exchange, we now turn our attention from the influential corpus of a classical French sociologist to a seminal publication of a contemporary French statistician. In 1965 in Toulouse, Henri Caussinus published an article on contingency tables in which the loglinear model of quasi-symmetry was the centerpiece. As to notation, with respect to a $g \times g$ table of counts, we will refer to an observed count in cell (i,j) as f_{ij}, and we will use F_{ij} to refer to the frequency in the same cell that is expected under some model. Under the model of symmetry, for all i and j, $F_{ij}=F_{ji}$. Among the ways Caussinus represented the model of quasi-symmetry was (in our notation)

$$\frac{F_{ij}}{F_{ji}}\frac{F_{jk}}{F_{kj}}\frac{F_{ki}}{F_{ik}}=1 \qquad (1)$$

(Caussinus, 1965, p. 147; see also Breiger and Ennis, 1997). For any i and j, defining F_{i+} as the sum of row i and F_{ij}/F_{i+} as the conditional probability $pr(j|i)$, then (1) implies

$$pr(j|i)\,pr(k|j)\,pr(i|k)=pr(i|j)\,pr(k|i)\,pr(j|k). \qquad (2)$$

In the extensive subsequent literature on quasi-symmetry, it is rare to see the model represented in either of these ways (Bishop, Fienberg and Holland, 1975, p. 287 provide one exception; see also footnote 8).

Now Caussinus certainly did not appear to consider himself a contributor to social exchange theory. Nonetheless, the implications of quasi-symmetry for generalized exchange are startlingly straightforward. Consider the $g \times g$ contingency table as expressing counts of interactions among group members. Under the model, all exchanges of the form $i \rightarrow j \rightarrow k \rightarrow i$ are equal in magnitude to flows in the opposite direction: $i \rightarrow k \rightarrow j \rightarrow i$. From this perspective, pure symmetry is a model of strong connections that are "brittle" (as in Ekeh, 1974, p. 52), while quasi-symmetry signifies a more robust and far-reaching interlocking of relations in the sense that the relevant flows in each triple of actors are directed "to" different people than they come "from."[6]

A principal motivation for the following discussion is that it is useful to provide new interpretations for existing statistical models, where possible, in order to harness their power for social network analysis and, in particular, for the study of social solidarity. Following the pathbreaking work of Sobel, Hout and Duncan (1985), whose principal concern was to provide a new interpretation for the model of quasi-symmetry in a substantive context – social mobility analysis – that is fairly distinct from the one that concerns us here, it has become customary to parameterize the quasi-symmetry model as follows:

$$F_{ij}=\alpha_j\beta_i\beta_j\delta_{ij}, \qquad (3)$$

where $\prod_j \alpha_j=1$, $\beta_i=\beta_j$ if $i=j$, $\delta_{ij}=\delta_{ji}$ if $i \neq j$, and $\delta_{ij}=1$ if $i=j$. Maximum likelihood procedures for estimation of the parameters and expected frequencies under this loglinear model are well-known (Bishop, Fienberg and Holland, 1975; Goodman, 1984; Agresti, 1990; Clogg and Shihadeh, 1994). Notice that Eq. (3) implies (1), so this parameterization allows us to maintain our focus on generalized exchange. An advantage of the Sobel–Hout–Duncan parameterization is that it distinguishes the symmetric components of the model (pairwise symmetry, the δ_{ij}, and marginal homogeneity, the β_j) from the asymmetric components, the α_j, which pertain in our context to the difference in volume between ties sent and ties received by any actor.

With respect to a pair of actors, say i and j, we may consider the extent to which each actor is oriented toward the other rather than

toward herself. For actor i, the odds are F_{ij}/F_{ii} that i chooses j rather than herself, and for actor j the odds are F_{ji}/F_{jj}. For both actors in this dyad, the average (geometric mean) of these odds is just δ_{ij}, as may be verified by substitution of Eq. (3); see also Sobel et al. (1985, p. 364):

$$\sqrt{\frac{F_{ij}}{F_{ii}}\frac{F_{ji}}{F_{jj}}}=\delta_{ij}$$

Each δ_{ij} is thus the average odds, estimated under the model, that actors i and j exchange with each other, rather than being self-sufficient. In the model of quasi-symmetry a possibly unique, symmetric δ parameter is required to be estimated for each pair of actors.

Our discussion of Durkheim's social morphology suggests that a more parsimonious version of quasi-symmetry may be formulated by identifying structured regions in a group's interaction table within which these pair-specific symmetries are equated. Suppose for example that, on *a priori* grounds, we have partitioned the set of g actors into B sets or "blocks," and that we apply this partition to both the rows and columns of the group interaction table of counts, such that the set of cells representing the interaction from actors in block k to those in block l is denoted B_{kl} (where k and l might index the same set). Then we may define the model of quasi-symmetry for "blocked" tables of counts of social interaction as Eq. (3) with the additional constraint that the symmetric δ_{ij} are constant for all cells $(i, j)\in B_{kl}$.

Until this point, we have discussed quasi-symmetry as a model of generalized exchange, and we have defined a more parsimonious version of quasi-symmetry that allows reduced-form ("blockmodel") representation of the interactional densities among pairs of actors. We now point out that quasi-symmetry, in both its usual and "blockmodel" versions, also implies the existence of an overarching hierarchy of actors that exists simultaneously with the pairwise patterning of ties and the possible presence of structured blocks of actors. For any actors i and j we may define the "excess" or "net" exchange from i to j as $R_{ij}=F_{ij}/F_{ji}$. (If i gives less to j than she receives, then the "net" exchange R_{ij} is less than unity.) Then under the model of quasi-symmetry Eq. (1) above may be rewritten as,

$$R_{ij}R_{jk}=R_{ik}$$

for any triples of actors i, j, k. Across all the actors in the group, there is a one-dimensional ordering with respect to "net" exchanges of the relationship in question. By substitution of Eq. (3) into the definition of R_{ij}, it is seen that $R_{ij}=\alpha_j/\alpha_i$, and the matrix $\mathbf{R}$ of size $g\times g$ whose typical entry is R_{ij} can be understood as composed of one dimension: $\mathbf{R}=\boldsymbol{uv}$ where $\boldsymbol{u}$ is a $g\times 1$ vector of entries $\langle 1/\alpha_1,\ldots,1/\alpha_g\rangle$ and $\boldsymbol{v}$ is a $1\times g$ vector of entries $\langle\alpha_1,\ldots,\alpha_g\rangle$.[7]

We think of the set of R_{ij} as analogous to an emergent "collective representation" of the group's hierarchy, in the following sense. If the model of quasi-symmetry fits the data, then in spite of whatever patterning the symmetric association takes among all the pairs of actors (as measured by the δ_{ij} parameters), the "net" magnitudes of exchange (the R_{ij}) form a one-dimensional hierarchy: the "net" exchange is precisely the ratio of the relative "strengths" (measured by the α_j parameters) of the two actors.[8] In a world governed by quasi-symmetry, it is as if all actors know, and universally agree upon, everyone's strength, including their own, so that the "net" exchanges among all pairs of actors depend on just g parameters, the α_j.

Because this single dimension composes the matrix $\mathbf{R}$ of net exchanges, the α_j are somewhat analogous to the economists' "structure of relative prices" that one contemporary theorist finds "very congenial to the structuralist approach" (Lindenberg, 1995, p. 85). Moreover, because the choice axiom of Luce (1959) provides a theoretical model of pairwise comparison processes that generates the hierarchy that is parameterized by our loglinear models (see footnote 8), it is incorrect of Skvoretz (1998) to assert that none is proposed. Indeed, this bridging of individual-level axiomatics and group properties – within a framework for empirical falsifiability of our claims – is a major feature exploited by our approach. More generally, we do not see why "theory" should be identified with individual-level processes, and on this point we recommend Markovsky's (1998) discussion of the "anthropocentrist trap."

One strategy that Lindenberg expounds for building bridges between structuralist and rational choice approaches is to develop "a theory of framing" to provide context for these "relative prices." We now turn to the articulation of some parameters that could well be relevant to the formulation and testing of such a theory.

The above feature of a collective representation of the group's hierarchical image, universally agreed upon, seems unduly rigid for many applications. In a manner that is at least loosely analogous to the formal model of self-location in a class system developed by Kosaka and Fararo (1991), we would like to incorporate our third theme from Durkheim, pertaining to the extent to which individuals

are capable of differentiating themselves from totalistic adherence to the collective representation of the group's hierarchy. For this purpose we consider a skew-symmetry model, which is (Yamaguchi, 1990, Eq. (21)) a natural generalization of quasi-symmetry:

$$F_{ij}=\alpha_j\beta_i\beta_j\delta_{ij}\epsilon_{ij}, \tag{4}$$

subject to the constraints of (3) and, in addition, for $i>j$, that $\epsilon_{ij}=\epsilon_j$ and $\epsilon_{ji}=1/\epsilon_j$; $\epsilon_{ij}=1$ if $i=j$; $\prod_j\epsilon_j=1$. Estimation of the ϵ_j requires $g-2$ nonredundant parameters beyond those estimated for quasi-symmetry; model (4) therefore leaves $(g-2)(g-3)/2$ degrees of freedom. Unlike quasi-symmetry, skew-symmetry requires the postulation of an ordering of the social actors.

For the skew-symmetry model (4), the quantity $(F_{ij}F_{ji}/F_{ii}F_{jj})^{1/2}$ is equal to δ_{ij}, just as it is in the quasi-symmetry model; this is the average odds that each member of a pair chooses the other, rather than oneself (other orientation vs. self-sufficiency). And as with quasi-symmetry, the analyst may modify (4) by stipulating that blocks of the δ_{ij} have identical magnitudes, thus obtaining a "blockmodel" version of skew-symmetry. The difference between (4) and (3) that becomes substantive in our discussion concerns the image of the group's hierarchy. Continuing to define R_{ij} as F_{ij}/F_{ji}, the "net" exchange between the actors, under model (4) we have, for $i>j$,

$$R_{ij}=\frac{\alpha_j}{\alpha_i}\epsilon_{ij}^2$$

thus introducing a person-specific ability to warp (i.e., to place greater or lesser emphasis upon) the one-dimensional, universally understood strengths. In other words, the network of "net" exchanges under skew-symmetry model (4) is conceptualized as two-dimensional, requiring strengths (α_j) and scores on a second dimension (ϵ_j) for each actor.

Characterization of this second dimension is of interest. Under the skew-symmetry model (4) we have

$$R_{ij}R_{jk}=R_{ik}\epsilon_j^2$$

for $i>j>k$. So for indirect relations which are monotone with respect to ordering, the middleman j has an actor-specific ability, constant across all appropriate i and k, to affect the magnitude of the exchange. In that this ability is a property of the actor "between" any two others (though not in all other respects), it is analogous to the "betweenness"

centrality of Freeman (1979; Freeman, Borgatti and White, 1991). We think of this parameter as indexing each actor's ability to thwart the group's hierarchy. At one extreme, an actor may be so committed to one part of the group that existence of a one-dimensional hierarchy is of little relevance to her. At the other extreme, an actor may bridge multiple segments of the group's morphological subdivisions, thus helping to interweave a network that confounds the hierarchy.

Markovsky (1998) criticizes us for not presenting "a metric for solidarity" as a means of quantifying approaches to or departures from such a state, and Skvoretz (1998) raises a similar concern. Therefore we state explicitly that the fit of our models may be assessed using standard maximum-likelihood and chi-square techniques, which do allow us to postulate a large family of substantive and relatively parsimonious models the fit of which to observed data may be assessed quantitatively (see also Breiger and Ennis, 1997). Following Durkheim's elaboration of a "sociological" approach to the study of solidarity, we reject perspectives which impose an assumption that solidarity is a scalar quantity. Nonetheless, we do exploit the available metric for assessing the degree of conformity of an observed structure to any of our models.

4. APPLICATIONS

Scholars who study decision-making in the U.S. Supreme Court have often relied on what Segal and Spaeth (1993) refer to as the legal model, which holds that Supreme Court decisions are based on the "plain meaning" of the Constitution, the intent of the framers, and precedent. Segal and Spaeth are among the leading researchers who investigate Supreme Court decision-making by use of an alternative framework, one which emphasizes the attitudes and values of the justices. We seek to study interagreement among Supreme Court justices as a network phenomenon amenable to analysis by the family of models that we have discussed. Our application is to a network of frequencies appearing as a table in Segal and Spaeth (1993, p. 280).

The data concern "special opinions" during the first four terms of the Rehnquist Court, 1986–1989. We include the eight justices who served continuously throughout this period, omitting two others: L. Powell and A. Kennedy.[9] The assignment of opinions by the chief justice to his associates is clearly a non-random process. For this and related reasons, Segal and Spaeth confine themselves to "special

opinions," defined as those which no justice can be forced to join or prevented from joining. (One example is the writing of an opinion notwithstanding membership in the majority or plurality opinion coalition; a more elaborate, technical definition is given in Segal and Spaeth, 1993, pp. 276–79.) Segal and Spaeth (p. 279) argue that writing and joining of special opinions "bespeak an ability to persuade or convince another of the correctness of one's position... without the use of coercion, authority, or political control."

The data in Table 8.1 are to be understood as in the following example. Justice Marshall joined Brennan in 92 special opinions that Brennan wrote (cell [1,2] in the first panel of Table 8.1 is 92) and did not join him in 21 others (cell [1,2] in the second panel). The total number of opinions Brennan wrote is $92+21=113$, reported as the second entry in the right-most column of Table 8.1. The proportion of times that Marshall joined Brennan is therefore $92/(92+21)=0.814$.

As a baseline model, we consider the elementary loglinear model that fits all two-way marginals of the $8\times8\times2$ array in Table 8.1, letting i refer to citing justice, j to cited justice, and k to citations given vs. not given. (Segal and Spaeth did not consider this network as a three-dimensional array, and therefore they did not control for the total number of opinions written by each justice, nor indeed did they consider application of loglinear models to their analytical concerns.)[10] Notation for this model, as well as test statistics, are reported in the first line of Table 8.2. In Table 8.2 the notation γ_{ij} refers to parameters necessary in order to insure that the sum of expected frequencies across the third dimension (indexed by k) exactly equals the corresponding sum of the observed frequencies, thus fitting the total number of opinions written by each justice. In addition to the degrees of freedom left by the model, the likelihood ratio chi-square (G^2), and the associated p-value, we report an alternative measure of model fit, the *bic* statistic of Raftery (1986), which is an effort to compensate for the overfitting that commonly results from extensive model search; large negative values of the *bic* statistic are to be preferred.

We can radically improve the fit of the baseline model at the expense of just one additional degree of freedom. Let us consider a partition of the eight justices, formulated *a priori*, into two blocks according to their typically understood ideological leanings as liberals (Marshall, Brennan, Stevens, Blackmun) or conservatives (O'Connor, White, Rehnquist, Scalia); see Simon (1995, pp. 14–16). We add one parameter to the baseline model, defined as follows and shown below with its

TABLE 8.1
Interagreement in Special Opinions, 1986–1989 terms (from Segal and Spaeth, 1993, p. 280)

	Frequency of opinion joining								Frequency of not joining								Number of opinions
Marshall	0	92	38	39	5	5	0	7	0	21	111	55	72	70	31	109	58
Brennan	46	0	33	40	6	6	0	6	12	0	116	54	71	69	31	110	113
Stevens	16	27	0	12	8	4	3	5	42	86	0	82	69	71	28	111	149
Blackmun	18	43	27	0	7	5	3	5	40	70	122	0	70	70	28	111	94
O'Connor	1	2	9	7	0	16	12	21	57	111	140	87	0	59	19	95	77
White	0	2	9	0	2	0	9	11	58	111	140	94	75	0	22	105	75
Rehnquist	0	0	8	1	19	22	0	24	58	113	141	93	58	53	0	92	31
Scalia	0	0	7	1	18	9	10	0	58	113	142	93	59	66	21	0	116

Special opinions are those in which no justice can be forced to, or prevented from, concurring or dissenting. Justices Powell and Kennedy are omitted as they did not serve throughout this period.

TABLE 8.2
Models Applied to Supreme Court Data of Table 1

Model	Parameterization	df	G^2	bic	p
All two-way effects	$F_{ij}=\alpha_{jk}\beta_{ik}\beta_{jk}\gamma_{ij}$	41	693.6	344	<0.001
Blockmodel Quasi-Symmetry (2 blocks)	$F_{ij}=\alpha_{jk}\beta_{ik}\beta_{jk}\gamma_{ij}\delta_{ij}$, $\delta_{ji}=\delta_{ij}=\delta_{qr}$ for $(i,j)\epsilon B_{qr}$	40	203.4	−137	<0.001
Quasi-Symmetry for 3 dimensions	$F_{ij}=\alpha_{jk}\beta_{ik}\beta_{jk}\gamma_{ij}\delta_{ij}$, $\delta_{ji}=\delta_{ij}$	21	33.8	−145	0.038
Skew-Symmetry for 3 dimensions	$F_{ij}=\alpha_{jk}\beta_{ik}\beta_{jk}\gamma_{ij}\delta_{ij}\epsilon_{ij}$, $\delta_{ji}=\delta_{ij}$ and $\epsilon_{ij}=\epsilon_j;\epsilon_{ji}=1/\epsilon_j$	15	19.1	−109	0.21

Blocks are (Marshall, Brennan, Stevens, Blackmun) vs. (O'Connor, White, Rehnquist, Scalia).

estimated value under maximum likelihood:

$$\begin{bmatrix} \delta_{ij} & \frac{1}{\delta_{ij}} \\ \frac{1}{\delta_{ij}} & \delta_{ij} \end{bmatrix} = \begin{bmatrix} 2.942 & .3399 \\ .3399 & 2.942 \end{bmatrix}.$$

Thus, all choices among the liberals, and all choices among the conservatives, are estimated to be 2.942 times the value they would have under the parameters of the baseline model, while all choices between these two groups are estimated to be only 1/2.942 as strong. Clearly this is a parsimonious version of quasi-symmetry (extended to the three-dimensional case) in which all symmetric interaction parameters are represented by a two-block blockmodel. As seen from a comparison of the first two lines of Table 8.2, this expenditure of one degree of freedom reduces the lack of fit by 71%(=1−203.4/693.6), and the *bic* criterion suggests no evidence of overfitting.

Moving from the blockmodel (line 2 of Table 8.2) to a natural three-dimensional representation of quasi-symmetry (line 3 of Table 8.2) involves estimating a unique δ_{ij} parameter for each pair of cells. The degrees of freedom left by this model is 20 less than that left by the baseline model (one df less than the number of pairs among 8–1 justices), but the lack of fit is also greatly reduced, leading to the most favorable *bic* value among models considered here and to a conventional level of fit ($p=0.038$) that is perhaps marginally acceptable.

In order to push the logic of this family of models, we consider in the fourth line of Table 8.2 a three-dimensional representation of our skew-symmetry model. This model requires postulation of an ordering of the actors, and the ordering we report is one of several that we explored.[11] The model fits very well by conventional standards ($p=0.21$). Estimates of parameters corresponding in the three-dimensional case to the δ parameters are reported in Table 8.3; similarly, estimates of the α and ϵ parameters are reported in Table 8.4.

The fine-grained patterning of the δ parameters in Table 8.3 is compatible with those estimated for the blockmodel (line 2 of Table 8.2) in the following sense. If we partition the justices in Table 8.3 into liberal and conservative blocks (the first four listed there vs. the last four), then the lowest δ_{ij} in the two submatrices reporting "within-group" propensities to join in opinions is 0.113, whereas the highest "between-group" propensity is only 0.080. In addition to this crucial cleavage within the structure of interactions that was captured with simplicity in the blockmodel of Table 8.2, however, more fine-grained detail is conveyed by the δ_{ij} of Table 8.3. For example, the liberals have a higher propensity to interact with one another than do the conservatives; in particular, the Brennan-Marshall tie is of extraordinary strength. This inward orientation of the liberals was during a period of apparent increasing conservative ascendancy (footnote 13; compare the nuanced account of Simon, 1995). The δ_{ij} also indicate that each of the conservatives has a higher propensity to join with Rehnquist than with anyone else, and this in spite of the formidable effort of Segal and Spaeth to eliminate from the dataset any special advantage accruing to the role of the chief justice.

Each of the entries in the 8 × 8 table **R** of net flows in Table 8.4 can be understood as the odds that a choice is made (rather than avoided)

TABLE 8.3
Estimated δ Parameters for Model of Skew-Symmetry (3 Dimensions)

Justice	$\delta^*=\{[F_{ij1}F_{ji1})/(F_{ii1}F_{jj1})]/[F_{ij2}F_{ji2})/(F_{ii2}F_{jj2})^1\}^{1/2}$							
Brennan	1.000	4.098	0.676	0.298	0.020	0.039	0.000	0.042
Marshall	4.098	1.000	0.571	0.344	0.024	0.026	0.000	0.035
Blackmun	0.676	0.571	1.000	0.190	0.019	0.024	0.034	0.079
Stevens	0.298	0.344	0.190	1.000	0.048	0.064	0.081	0.080
Scalia	0.020	0.024	0.019	0.048	1.000	0.120	0.381	0.254
White	0.039	0.026	0.024	0.064	0.120	1.000	0.369	0.113
Rehnquist	0.000	0.000	0.034	0.081	0.381	0.369	1.000	0.455
O'Connor	0.042	0.035	0.079	0.080	0.254	0.113	0.455	1.000

TABLE 8.4
Net Flows R*, and Estimated α and ϵ Parameters, for model of skew-symmetry (3 dimensions)

Justice	R* = $(F_{ij1}/F_{ji1})/(F_{ij2}/F_{ji2})$							Justice	α_j^*	Justice	ϵ_j^2
Brennan	1.000	0.875	1.267	0.971	5.424	4.924	8.624	3.434	Rehnquist	3.136	Stevens	3.972
Marshall	1.143	1.000	1.462	1.120	6.258	5.682	9.950	3.962	Scalia	1.973	White	2.864
Blackmun	0.789	0.684	1.000	1.070	5.977	5.427	9.504	3.785	White	1.791	O'Connor	1.000
Stevens	1.030	0.893	0.934	1.000	1.406	1.277	2.236	0.890	Brennan	1.396	Marshall	0.991
Scalia	0.184	0.160	0.167	0.711	1.000	1.467	2.569	1.023	O'Connor	1.249	Rehnquist	0.768
White	0.203	0.176	0.184	0.783	0.682	1.000	0.611	0.243	Blackmun	0.461	Blackmun	0.716
Rehnquist	0.116	0.101	0.105	0.447	0.389	1.636	1.000	0.519	Stevens	0.353	Scalia	0.619
O'Connor	0.291	0.252	0.264	1.123	0.978	4.108	1.928	1.000	Marshall	0.318	Brennan	0.261

$\alpha_j^* = (\alpha_{j1}/\alpha_{i1})/(\alpha_{j2}/\alpha_{i2})$.

from actor i to j, in ratio to the odds that it is made (rather than avoided) in the opposite direction. This matrix of "net intensity" of flows is, according to the skew-symmetry model, capable of decomposition into two dimensions: an overall "strength" of each justice (the aj) and the extent to which each justice mediates flows among others in a sense roughly analogous to Freeman's "betweenness" (the ϵ_j parameters). Consider the "net intensity" of the tie from O'Connor (i) to Scalia (k). This value, 0.978 from matrix **R** in Table 8.4, is equal to (1.973/1.249) (0.619) = $(\alpha_k/\alpha_i)(\epsilon_k)$; see the right-hand side of Table 8.4. Consider also the extent to which the net exchange from O'Connor to Scalia is mediated through Byron White (j). This is the value of the net exchange from O'Connor to White ($R_{ij}=4.108$) times that from White to Scalia ($R_{jk}=0.682$) times the extent to which Byron White affects the flow ($1/2.864=1/\epsilon_j$), which is also equal to 0.978 ($=R_{ik}$). Moreover, according to the model Byron White affects the flow between any two actors to exactly the same extent, $1/\epsilon_j$.

We substantively interpret the dimensional parameters in Table 8.4 as follows. First, the "strengths" (α_j) make a good deal of sense. Rehnquist, the chief justice, emerges with the highest score, a finding which perhaps calls into question the ease with which even a dataset confined to "special opinions" can be successful in eliminating the influence of the "first among equals." The next four highest "strengths" include the other conservatives on the court plus Brennan, who is described as the leader of the liberal wing (Simon, 1995, p. 14).

The skew-symmetry parameters estimated in Table 8.4 are of particular interest. The two lowest ϵ_j scores are for the two most ideological justices: Brennan, the leader of the liberal wing, and Scalia, the Court's most aggressive conservative. At the other end of this pole, the highest ϵ_j are those of Stevens, White, and O'Connor, all of whom could well be argued to be "swing" voters. For example, Stevens is often characterized as a liberal but has "confounded a media eager to place the justices into neatly defined categories" (Simon, 1995, p. 15); Stevens joined the conservatives in opposing a right for protesters to burn the flag, and he was described by Blackmun as "a maverick, imaginative."[12] As to Byron White and O'Connor, one or more of these conservatives voted with the liberals in all three of the 22 cases in which the liberals voted as a block and prevailed during the term ending in June 1989.[13]

With reference to our third theme from Durkheim, we would say that departures from the one-dimensional set of strengths governing net exchanges under quasi-symmetry are due to two extremes that are

identified by the "second" dimension that appears in the estimated parameters from skew symmetry: on the one hand extreme commitment to a subgroup (as in the cases of Scalia and Brennan), and on the other hand extreme commitment to individual courses of action (as in the case of the "swing" voters among the justices).

5. CONCLUSION

We have used social network thinking to create a family of models that build directly on the work of Durkheim. Our concern is focused on the morphology of social association and the collective representation of group hierarchy. We model solidarity as part of the emergence of morality through generalized exchange in social collectivities.

All four of the foci identified in Durkheim's treatment of solidarity were seen to be applicable to the modeling framework that was brought to bear in analyzing interagreement among Supreme Court justices. Detailed pairwise relationships were largely captured in a simplified macro-structure. These pair-specific symmetric exchanges were fully compatible with a hierarchical set of "strengths" that is universal in the quasi-symmetry model in the sense that "net" exchanges are functions of ratios of strengths of the actors (one dimension). In the skew-symmetry model, "net" exchanges are two-dimensional, the second dimension pertaining to ideological commitments to the group or to a segment of the group (at one extreme) and to the ability to weave coalitions together, making or breaking them (at the other extreme).

It seems remarkable that such different notions of structure – a detailed patterning of symmetric interpersonal interaction within the group, possibly represented by a much simpler blockmodel image, but also a rigid hierarchy of "net" exchange captured by one dimension (quasi-symmetry) or two (skew-symmetry) – can coexist within the same social group, and indeed within the same model of that group. And yet, it may be argued that a major result of network analysis is the recognition of a potential multiplicity of equivalences that can be mobilized so as to complement each other in analyzing social structure (Doreian, 1988). In the context of social mobility studies, analysts have recognized the different forms of structuring that are captured in quasi-symmetry ("exchange" and "structure," in the terminology of Sobel et al., 1985) and in skew-symmetry (Yamaguchi, 1990). In this paper we have developed a parallel interpretive framework for making feasible the application of these models to the study of solidarity in social networks that arise in the form of frequency data. It will continue to be important, in further work that brings together concerns of loglinear modelers and network analysts, to be aware of different organizing principles such as those identified here, and to capture various organizing principles within the same model or family of models.

NOTES

1. Skvoretz (1998) criticizes us for giving no indication of how "the group's level of solidarity shapes... patterns and structures" of group interaction, but he provides no justification for his implicit assumption that solidarity is a scalar quantity. In viewing solidarity as morphological, we follow the theoretical analysis of Durkheim. In that each of our models (Table 2) provides an ideal-type image of group structure, these models bear interesting relations to the "referent networks" sought by Markovsky (1998). We also provide a statistical context for relating observed network data to referent network imagery.
2. Lindenberg (1995, pp. 84–85) faults both the early Durkheim and contemporary structural analysts for "this innocence with regard to a theory of action." Similar epithets are hurled in the opposite direction (see the chapter on "Rhetoric and Theory" in White, 1992, pp. 287–316). Both camps are sites of important analytical insights and models, and the distance between them is occasionally exaggerated. Indeed, as we briefly mention in Section 3 of this paper, the modeling framework that we articulate can be developed as yet another bridge between rational choice and structuralist analyses.
3. More recently, Frey (1960) has advocated so-called "objective" definitions of democratic, autocratic, and laissez-faire structures, based on the mathematics of graph theory. Application of Frey's idealized models to real data of any size would prove impractical, however, due to the small likelihood that any actual group would conform precisely to any of them. See Breiger (1991, pp. 98–175) for a blockmodel approach to analysis of governance structures.
4. Just as a blockmodel presupposes homogeneous blocks and then describes their interrelations, so Ralph Turner has argued that organic solidarity does not replace, but "requires, in addition to the division of labor, an effective substratum of mechanical solidarity" (quoted in Ekeh, 1974, p. 66).
5. The points at issue in the on-going debate between culturalist interpreters of Durkheim (see the collection edited by Alexander, 1988) and structuralists (seemingly at times everyone else) are, first, the extent to which the emergent culture of representations becomes autonomous from the social structure, and, second, the extent to which there was a great divide between Durkheim's own early and later writings on this topic. We are persuaded, however, by Collins' argument that the ingredients of Durkheim's theory of solidarity are in all his major works, early and late, and that, in its entirety, Durkheim's corpus challenges us to elaborate "principles of how different degrees and conditions of structural density of interaction result in degrees of 'moral density' and hence in different kinds of symbolic and emotional consciousness of the group" (Collins, 1988, pp. 110–11).
6. Extensions of generalized exchange to models for exchanges among quartets (or more) of actors are implied by models of skew-symmetry such as the one discussed later in this section; see also Bishop et al. (1975, p. 291).
7. Moreover, with respect to inner-product matrix multiplication used to compute powers, the matrix $\mathbf{R}^k = ug^{k-1}v$.

8. This discussion of R_{ij} is somewhat analogous to related formulations. Luce (1959) brings concepts of strong and weak stochastic transitivity to bear in elucidating implications of the choice axiom for the scaling of preferences. Theorem 2 in Luce (1959, pp. 16–17) concerns symmetry of intransitivity among triples of choices and is similar to the focus of our Eqs. (1) and (2) on symmetry among triples of actors. The Luce model implies a linear preference model in the logit scale (Fienberg and Larntz, 1976, p. 248), and aspects of Luce's model, as well as the Bradley–Terry model for paired comparison experiments, have been formulated in terms of the quasi-symmetry model by Fienberg and Larntz (see also discussions in Agresti, 1990, pp. 370–74, and in Roberts, 1990, pp. 85–86). In a very different substantive context, in their discussion of "structural mobility" Sobel et al. (1985) also imply the one-dimensionality upon which we focus with respect to the network of the R_{ij}.
9. We have performed analyses similar to those reported below on data for all ten justices (including Powell and Kennedy); the results for the eight justices included here are very similar. We confine ourselves here to the eight justices present throughout the observation period in order to insure that all justices we include have the same chance to join one another in opinions.
10. Representation of a square, $g \times g$ network of counts by means of a three-dimensional, $g \times g \times 2$ array (where the third dimension indexes ties given versus ties withheld) is formally analogous to the problem of modeling blockmodel densities that was first considered with respect to loglinear models by Marsden (1989), whose model (9) is identical to our baseline. This topic is further considered in Breiger and Ennis (1997).
11. The ordering we imposed is {Brennan, Marshall, Blackmun, Stevens, Scalia, White, Rehnquist, O'Connor}. The first four would be conventionally labeled "liberal" and the second four as "conservative." In this sense the ordering is a more fine-grained version of the two-block model of line 2 of Table 8.2.
12. *New York Times* of June 22, 1989 and July 18, 1988, respectively.
13. Linda Greenhouse, "News analysis: The year the court turned right," *New York Times*, July 7, 1989.

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